

Cavity QED effects in semiconductor microcavities

H Eleuch, J M Courty, G Messin, C Fabre and E Giacobino

Laboratoire Kastler Brossel, Université Pierre et Marie Curie, Ecole Normale Supérieure et CNRS, case 74, 4 place Jussieu, F-75252 Paris Cedex 05, France

Received 31 July 1998, in final form 5 November 1998

Abstract. A theoretical investigation of cavity QED effects in semiconductor microcavities containing quantum wells is presented. A model Hamiltonian is used to derive equations of motion for the quantum photon and exciton fields in the cavity. Quantum effects such as squeezing and antibunching are predicted in the light field going out of the cavity under irradiation by a coherent laser field, if exciton–phonon scattering is weak enough. Exciton–phonon scattering is shown to destroy the nonclassical effects and to yield excess noise in the output field.

Keywords: Quantum noise, squeezing, antibunching, microcavities, semiconductor quantum wells, excitons

1. Introduction

Semiconductor microcavities offer several new attractive features and have been the subject of numerous publications since the demonstration of the possibility to reach the strong coupling regime in such systems [1]. Strong coupling is observed when the cavity mode is nearly resonant with a narrow optical transition of the active medium. If the coupling frequency corresponding to a single photon is larger than the relaxation frequencies of the medium and of the cavity, the so-called vacuum Rabi splitting is observed. The degeneracy between the cavity resonance and the medium resonance is lifted and two lines appear in the reflection or in the transmission spectrum of the system. Semiconductor quantum wells exhibit narrow absorption lines corresponding to excitonic resonances. The large oscillator strengths of these resonances make it possible to achieve the strong coupling regime and to observe vacuum Rabi splitting. The strong coupling between the exciton and the cavity can equivalently be interpreted in the cavity polariton model [2–4].

While the quantum properties of the light field going out of microcavities containing atoms in the strong coupling regime have been investigated by several authors in the context of QED [5–8], semiconductor microcavities have been the subject of very few studies in this respect [9, 10]. It is the purpose of this paper to present theoretical predictions concerning QED effects observable in semiconductor microcavities in the strong coupling regime.

At this point it must be noted that the presence of strong coupling is not a quantum feature in itself since it appears in the coupling of two classical oscillators such as pendulums or in the coupling of a cavity with a set of classical dipoles [11, 12]. As a matter of fact, it is well known that the linear coupling of the electromagnetic field with another harmonic

oscillator, which is easily diagonalized in two eigenstates, leads to a linear equation of motion and, as such, cannot modify the quantum properties of the field. In the case of cavities containing atoms, the specifically quantum features, that are revealed by the generation of nonclassical states of the field in the cavity [8, 13, 14], arise from the coupling of the field with the atomic dipole, which is basically nonlinear.

In the case of semiconductors, the interaction with the field occurs via excitons, which are often modelled as harmonic oscillators. However, semiconductors exhibit nonlinear properties due to Coulomb interaction and Pauli exclusion between carriers and to band filling [15]. For these nonlinearities to be of interest for quantum optics, they have to give rise to coherent effects. In spite of numerous nonradiative relaxation processes that cause a fast decay of coherences, semiconductors have already been shown to generate coherent nonlinear effects [16], such as dynamical Stark shift [17]. Furthermore, recent experiments have demonstrated the possibility to modify the quantum fluctuations and to generate squeezing in semiconductors [18].

In this paper, we use a model Hamiltonian that includes a nonlinear interaction between excitons to deal with the system. Relaxation and dissipation effects are accounted for in a phenomenological way in the equation of motion. This allows us to predict quantum effects in the light field going out of the semiconductor microcavity. The model is not meant to describe accurately all the interactions taking place in the semiconductor. Nevertheless, we expect that it gives the main features of the observable quantum phenomena.

2. Model for the interaction

We consider a microcavity made of two highly reflecting planar mirrors spaced by a distance which is of the order

of the wavelength. A thin layer of semiconductor quantum well parallel to the mirrors is placed in the middle of the microcavity (or at an antinode of the field). The discussion is limited to a two-band semiconductor. The electromagnetic field can excite an electron from the filled valence band to the conduction band, thereby creating a hole in the valence band. The electron–hole system possesses bound states, the excitonic states, analogous to the hydrogenic states, or more precisely to the positronium bound states. We will only consider the first of these bound states, the 1s state.

The interaction between a semiconductor quantum well and the field in a microcavity can be modelled by the coupling of two harmonic oscillators, together with some excitonic nonlinearity. The Hamiltonian of the coupled system is written as

$$H = \hbar\omega_{cav}\hat{a}^\dagger\hat{a} + \hbar\omega_{exc}\hat{b}^\dagger\hat{b} + \hbar g_1(\hat{a}^\dagger\hat{b} + \hat{b}^\dagger\hat{a}) + \hbar\alpha_1\hat{b}^\dagger\hat{b}^\dagger\hat{b}\hat{b}. \quad (1)$$

The first two terms correspond to the energies of the cavity field and of the excitonic field: $\hat{a}$ and $\hat{b}$ are respectively the annihilation operators of a photon and of an exciton in the cavity, ω_{cav} and ω_{exc} are the frequencies of the cavity resonance and exciton resonance.

The third term corresponds to the exciton–photon coupling, the strength of which is proportional to g_1 . The last term describes the nonlinearity arising from Coulomb interaction between excitons and from Pauli exclusion effects and may be modelled by a Kerr-type excitonic nonlinearity [15]. The other nonlinear term appearing at the same order in the fields and due to saturation effects will be neglected here. It can be shown that it gives rise to small corrections compared with the previous one [19].

Due to the translational invariance in the plane of the semiconductor layers, excitons with a wavevector $K_{\parallel}$ in this plane can only be coupled with light having an equal wavevector $k_{\parallel}$. On the other hand, in the direction perpendicular to the layers, the exciton modes are quantized, as well as the cavity modes. Subsequently, we can consider the interaction of a cavity mode with one exciton mode only, giving rise to strong coupling. We shall assume that all the other exciton modes form a thermal reservoir that is weakly coupled to the modes of interest.

This Hamiltonian is similar to the one studied in [20] leading to the prediction of quantum effects in propagation without a cavity. On the other hand, there has been a broad literature describing squeezing in cavities containing a nonlinear medium close to the bistability threshold, in the weak coupling regime [21–24]. The squeezing properties of the field going out of a cavity containing atoms in the strong coupling regime were investigated in [13, 25].

Here we compute the squeezing spectra and the second-order correlation function of the output field of a microcavity containing excitons, for which the nonlinearity is different from the atomic one and necessitates a specific treatment. We emphasize the case of very strong coupling (where the vacuum Rabi splitting is much larger than the cavity and material decay rates), which is often encountered in such systems and has not been investigated in detail previously. Moreover, we study the effect of the presence of a thermal reservoir of excitons coupled to the system.

The microcavity is irradiated by a coherent field from a laser of frequency ω_L . One mirror of the microcavity is perfectly reflecting, whereas the other one, having a small non-zero transmission coefficient, is the coupling mirror, through which the light is coupled in and out. Using the Hamiltonian and including the relaxation processes, one can derive evolution equations for the time-dependent electromagnetic field and exciton field operators inside the cavity:

$$\frac{d\hat{a}}{dt} = -(\gamma_a + i\delta_a)\hat{a} - ig\hat{b} + \sqrt{2\gamma_a}\hat{a}^{in} \quad (2a)$$

$$\frac{d\hat{b}}{dt} = -(\gamma_b + i\delta_b)\hat{b} - ig\hat{a} - i\alpha\hat{b}^\dagger\hat{b}\hat{b} + \sqrt{2\gamma_b}\hat{b}^{in} \quad (2b)$$

where t is a dimensionless time normalized to the round trip time τ_c in the cavity, γ_a and γ_b are the dimensionless decay constants of the field in the cavity and of the exciton, i.e. the cavity field and exciton decay rates normalized to $1/\tau_c$. The other parameters, δ_a and δ_b , the detunings of the cavity and of the exciton frequencies from the frequency ω_L of the incoming laser field ($\hat{a}^{in}$), g the exciton to photon coupling constant and α the nonlinear coupling constant are also normalized to $1/\tau_c$

$$\delta_a = (\omega_{cav} - \omega_L)\tau_c \quad \delta_b = (\omega_{exc} - \omega_L)\tau_c \quad (3a)$$

$$g = g_1\tau_c \quad \alpha = 2\alpha_1\tau_c. \quad (3b)$$

The fields $\hat{a}^{in}$ and $\hat{b}^{in}$ are the incoming electromagnetic and exciton fields (coherent or thermal fields). The mean values of the field intensities will be expressed in numbers of particles per unit time. As we have chosen the unit time to be the cavity round trip time, these mean values are numerically equal to the average numbers of particles in the cavity.

The decay constant γ_a of the field in the cavity is related to the amplitude reflection coefficient r of the coupling mirror by $r = 1 - \gamma_a$. Since the cavity has a high finesse, r is close to one, which implies that the amplitude transmission coefficient t is much smaller than one and verifies $t = \sqrt{2\gamma_a}$.

The decay constant γ_b of the exciton is due to two processes: the electron–hole radiative recombination occurring on defects of the quantum well (that do not conserve $K_{\parallel}$) and the decay towards other exciton modes (modes with different values of $K_{\parallel}$). The coupling with other exciton modes mainly occurs through the acoustic phonons. In this process, an exciton in the mode of interest is annihilated while an exciton in a reservoir mode is created and a phonon is created or annihilated [26]. We neglect the relaxation due to direct collisions between excitons in the mode of interest and excitons in the other modes, which is less efficient at the excitonic densities considered here [27–29]. Let us note that when the coupling through phonons decreases (for example at very low temperature, or if some phonon modes are forbidden in the quantum wells [30]), the redistribution of exciton population from the resonant mode to all other modes is less efficient and scattering between excitons in different modes [27] decreases accordingly.

We solve the problem in the framework of the input–output formalism [31], where the evolution of the fields for the electromagnetic and exciton modes is computed using

equations (2) while the output field is related to the intracavity and input fields by [32]:

$$\hat{a}^{out} = \sqrt{2\gamma_a}\hat{a} - \hat{a}^{in}. \quad (4)$$

Equation (4) indicates that the outgoing field is the sum of the inside field transmitted through the coupling mirror and of the input field reflected by the mirror (the reflection coefficient r has been replaced by 1 in this equation).

Let us note that equations (2) can alternatively be considered as Langevin equations for the two fields, where the fluctuating part of the terms $\sqrt{2\gamma_a}\hat{a}^{in}$ and $\sqrt{2\gamma_b}\hat{b}^{in}$ are the Langevin forces associated with the reservoirs for the electromagnetic field and for the excitons.

The incoming electromagnetic field $\hat{a}^{in}$ is the laser coherent field, which is a classical field together with quantum fluctuations equal to the vacuum fluctuations. The exciton field inside the cavity is coupled with a reservoir, represented by the fluctuating field $\hat{b}^{in}$. We consider two cases, the first where the reservoir is at zero temperature, $\hat{b}^{in}$ is then the vacuum field, and the second where the reservoir is a thermal bath the temperature of which depends on the experimental conditions. Thus for the electromagnetic field, we have

$$\hat{a}^{in} = a^{in} + \delta\hat{a}^{in} \quad (5)$$

where a^{in} is the classical mean value of the field and where its fluctuations $\delta\hat{a}^{in}$ have a zero mean value $\langle\delta\hat{a}^{in}\rangle = 0$. Their correlation functions are given by

$$\langle\delta\hat{a}^{in}(t)\delta\hat{a}^{in\dagger}(t')\rangle = \delta(t - t') \quad (6a)$$

$$\begin{aligned} \langle\delta\hat{a}^{in\dagger}(t)\delta\hat{a}^{in}(t')\rangle &= \langle\delta\hat{a}^{in}(t)\delta\hat{a}^{in}(t')\rangle \\ &= \langle\delta\hat{a}^{in\dagger}(t)\delta\hat{a}^{in\dagger}(t')\rangle = 0. \end{aligned} \quad (6b)$$

For the exciton field, the incoming field has a zero mean value

$$\hat{b}^{in} = \delta\hat{b}^{in} \quad (7)$$

with $\langle\delta\hat{b}^{in}\rangle = 0$. The correlation functions of $\delta\hat{b}^{in}$ are given by

$$\langle\delta\hat{b}^{in}(t)\delta\hat{b}^{in\dagger}(t')\rangle = (1 + \langle n \rangle)\delta(t - t') \quad (8a)$$

$$\langle\delta\hat{b}^{in\dagger}(t)\delta\hat{b}^{in}(t')\rangle = \langle n \rangle\delta(t - t') \quad (8b)$$

$$\langle\delta\hat{b}^{in}(t)\delta\hat{b}^{in}(t')\rangle = \langle\delta\hat{b}^{in\dagger}(t)\delta\hat{b}^{in\dagger}(t')\rangle = 0. \quad (8c)$$

$\langle n \rangle$ is the mean number of excitations in the thermal bath. We assume that the coupling occurs mainly with the reservoir of nonradiative excitons whose energy differs from the energy of the studied mode by ΔE , via absorption or emission of phonons [4, 30]. The phonons needed to match the energy difference ΔE have a mean occupation number $\langle n \rangle$:

$$\langle n \rangle = \frac{1}{\exp(\Delta E/kT) - 1}. \quad (9)$$

For the cases of interest ΔE is of the order of half the vacuum Rabi splitting, corresponding to $\Delta E/kT = 1$ for $T \sim 25$ K†.

† If the damping associated with direct exciton–exciton collisions is small compared with other damping rates, the noise associated to this damping can be shown to be also small using the derivation of Walls and Milburn [33].

3. Mean fields

We first rewrite equations (2) for classical values of the fields, suppressing the fluctuating terms, and we solve them in the steady state regime in order to study the mean values of the electric field. The results of the calculation are presented for cases in which the exciton is resonant with the cavity. Due to the strong coupling, the degeneracy between the cavity and the exciton resonances is lifted and two symmetrical peaks appear in the variation of the electromagnetic and exciton fields as a function of the laser frequency. When nonlinear processes are taken into account, changes are found in the shape of the peaks.

Figures 1 and 2 show the variation of the intracavity electromagnetic field intensity with laser detuning when exciton and cavity are in resonance, respectively for moderately strong coupling and for very strong coupling. We have taken equal cavity and exciton widths, $\gamma_a = \gamma_b = 0.25g$ in the first case and $\gamma_a = \gamma_b = 0.05g$ in the second case. The photon–exciton coupling coefficient g is equal to 2×10^{-2} in units of the inverse round trip time in the microcavity. In the same units, the nonlinear coefficient α was evaluated from [15] to be 1.5×10^{-9} for an active area of 0.1 mm^2 .

The intracavity electromagnetic field intensities are expressed in units of a *saturating* intensity I_0 . I_0 is defined as the intracavity electromagnetic intensity yielding a density of excitons of $10^9 \text{ excitons cm}^{-2}$ over the active area of 0.1 mm^2 in the absence of nonlinear effects. The latter value is usually considered as the limit of the low-density case [34], where our treatment is expected to be valid. Beyond this value, higher-order effects in the exciton–exciton interaction take place and free-carrier screening eventually bleaches the excitonic oscillator strength.

Figure 1(a) shows the variation of the intracavity field intensity in the moderately strong coupling regime and in a case where the maximum intensity I_m in the absence of nonlinear effects is $I_m = 1.6I_0$. It can indeed be seen that in spite of a small asymmetry between the two peaks, the calculated spectrum is compatible with experimental observations. The value of the intensity in figure 1(b) is four times as large, $I_m = 6.4I_0$, and one of the peaks starts to exhibit a steep edge, that has never been observed experimentally. This result is unrealistic since it corresponds to excitations for which the excitons are saturated. Actually, experiments showed that in such cases the bleaching of the oscillator strength causes the strong coupling effect to disappear [34, 35].

In figure 2 we show the case of very strong coupling and two values of the input intensity. The value of the intracavity field in figure 2(a) is below saturation, $I_m = 0.5I_0$, while in figure 2(b) it is $I_m = 2I_0$. The shape of the curves in these cases are again compatible with experimental observations. For the calculation of the squeezing effects, we will focus on the case of weak nonlinear effects, in which the peaks are not too much changed from their shapes in the absence of nonlinearity, i.e. we will use the experimental parameters of figures 1(a), 2(a) and 2(b).

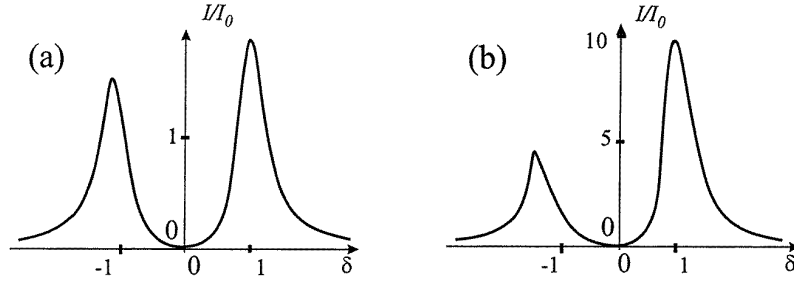


Figure 1. (a) Intensity I of the cavity mode as a function of the laser detuning (normalized to g) when the cavity is resonant with the exciton and for a nonlinear parameter $\alpha = 1.5 \times 10^{-9}$. The other parameters are as follows: losses $\gamma_a = \gamma_b = 0.5 \times 10^{-2}$, exciton-photon coupling $g = 2 \times 10^{-2}$; maximum intracavity intensity in the absence of nonlinearity, $I_m = 1.6I_0$. (b) The same as (a) with a stronger excitation, $I_m = 6.4I_0$.

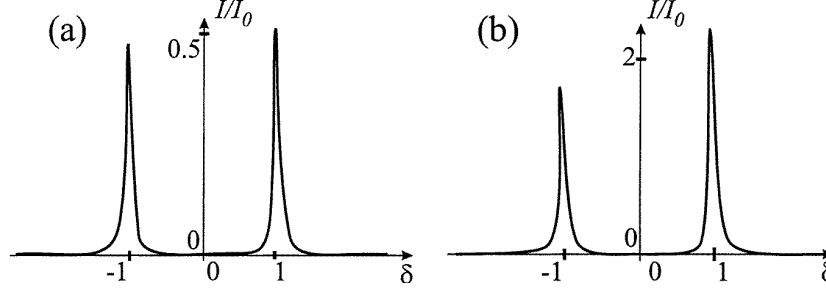


Figure 2. Intensity I of the cavity mode as a function of the laser detuning δ (normalized to g) when the cavity is resonant with the exciton with the same parameters as in figure 1, but in the very strong coupling regime: losses $\gamma_a = \gamma_b = 0.1 \times 10^{-2}$, maximum intracavity intensity in the absence of nonlinearity, $I_m = 0.5I_0$ for (a) and $I_m = 2I_0$ for (b).

4. Modifications of quantum fluctuations

We will deal here with the case in which the electromagnetic and exciton fields have large average values compared with the fluctuations. We can then linearize equations (2) using

$$\hat{a}(t) = a_0 + \delta\hat{a}(t) \quad (10a)$$

$$\hat{b}(t) = b_0 + \delta\hat{b}(t) \quad (10b)$$

where a_0 and b_0 are the (classical) mean values derived in the previous section and $\delta\hat{a}(t)$ and $\delta\hat{b}(t)$ are the quantum fluctuations of the field inside the cavity that we are going to compute.

We will use the Fourier transforms of the fluctuations defined as

$$\delta\hat{a}[\omega] = \int \delta\hat{a}(t) e^{i\omega t} dt \quad (11a)$$

$$\delta\hat{a}^\dagger[\omega] = \int \delta\hat{a}^\dagger(t) e^{i\omega t} dt \quad (11b)$$

where ω is a dimensionless frequency normalized to the inverse round trip time in the cavity.

Linearizing equations (2) and taking their Fourier transform, we obtain:

$$-i\omega\delta\hat{a}[\omega] = -(\gamma_a + i\delta_a)\delta\hat{a}[\omega] - ig\delta\hat{b}[\omega] + \sqrt{2\gamma_a}\delta\hat{a}^{in}[\omega] \quad (12a)$$

$$-i\omega\delta\hat{b}[\omega] = -(\gamma_b + i\delta_b)\delta\hat{b}[\omega] - ig\delta\hat{a}[\omega] - 2i\alpha|b_0|^2\delta\hat{b}[\omega] - i\alpha b_0^2\delta\hat{b}^\dagger[\omega] + \sqrt{2\gamma_b}\delta\hat{b}^{in}[\omega] \quad (12b)$$

and similar equations for the complex conjugates. These equations can be written in a matrix form as

$$-i\omega \begin{bmatrix} \delta\hat{a}[\omega] \\ \delta\hat{a}^\dagger[\omega] \\ \delta\hat{b}[\omega] \\ \delta\hat{b}^\dagger[\omega] \end{bmatrix} = D \begin{bmatrix} \delta\hat{a}[\omega] \\ \delta\hat{a}^\dagger[\omega] \\ \delta\hat{b}[\omega] \\ \delta\hat{b}^\dagger[\omega] \end{bmatrix} + T \begin{bmatrix} \delta\hat{a}^{in}[\omega] \\ \delta\hat{a}^{in\dagger}[\omega] \\ \delta\hat{b}^{in}[\omega] \\ \delta\hat{b}^{in\dagger}[\omega] \end{bmatrix} \quad (13)$$

where D is the drift matrix

$$D = - \begin{bmatrix} \gamma_a + i\delta_a & 0 & ig & 0 \\ 0 & \gamma_a - i\delta_a & 0 & -ig \\ ig & 0 & \gamma_b + i\delta_b + 2i\alpha|b_0|^2 & i\alpha b_0^2 \\ 0 & -ig & -i\alpha b_0^2 & \gamma_b - i\delta_b - 2i\alpha|b_0|^2 \end{bmatrix} \quad (14)$$

and T is the transmission matrix

$$T = \text{Diag} [\sqrt{2\gamma_a}, \sqrt{2\gamma_a}, \sqrt{2\gamma_b}, \sqrt{2\gamma_b}]. \quad (15)$$

The solution of equations (12) is then given by

$$[\delta\hat{a}[\omega]] = G[\omega]T[\delta\hat{a}^{in}[\omega]] \quad (16)$$

where $[\delta\hat{a}[\omega]]$ and $[\delta\hat{a}^{in}[\omega]]$ are the column vectors appearing in equation (13) and representing the fluctuations of the inside fields and of the input fields and where $G[\omega]$ can be calculated from

$$(G[\omega])^{-1} = -i\omega \text{Id} - D[\omega] \quad (17)$$

where Id is the identity matrix.

Using equation (4), the output field fluctuations are given by

$$[\delta\hat{a}^{out}[\omega]] = -[\delta\hat{a}^{in}[\omega]] + T[\delta\hat{a}[\omega]] \quad (18)$$

that is

$$[\delta\hat{a}^{out}[\omega]] = [-\text{Id} + TG[\omega]T][\delta\hat{a}^{in}[\omega]]. \quad (19)$$

Solving these equations allows us to calculate noise power spectra in the output field such as $S_{\hat{a}\hat{a}}^{out}$ and $S_{\hat{a}\hat{a}^\dagger}^{out}$

$$\langle \delta \hat{a}^{out}[\omega] \delta \hat{a}^{out}[\omega'] \rangle = 2\pi S_{\hat{a}\hat{a}}^{out}[\omega] \delta[\omega + \omega'] \quad (20a)$$

$$\langle \delta \hat{a}^{out}[\omega] \delta \hat{a}^{out\dagger}[\omega'] \rangle = 2\pi S_{\hat{a}\hat{a}^\dagger}^{out}[\omega] \delta[\omega + \omega']. \quad (20b)$$

For this purpose we use the noise spectra of the input fields derived from equations (6) and (8), that are white noise spectra due to the fact that the fluctuations of the input fields are δ correlated.

$$S_{\hat{a}\hat{a}^\dagger}^{in}[\omega] = 1 \quad (21a)$$

$$S_{\hat{a}\hat{a}}^{in}[\omega] = S_{\hat{a}^\dagger\hat{a}^\dagger}^{in}[\omega] = S_{\hat{a}^\dagger\hat{a}}^{in}[\omega] = 0 \quad (21b)$$

$$S_{\hat{b}\hat{b}^\dagger}^{in}[\omega] = 1 + \langle n \rangle \quad (21c)$$

$$S_{\hat{b}^\dagger\hat{b}}^{in}[\omega] = \langle n \rangle \quad (21d)$$

$$S_{\hat{b}\hat{b}}^{in}[\omega] = S_{\hat{b}^\dagger\hat{b}^\dagger}^{in}[\omega] = 0. \quad (21e)$$

Experiments allow us to measure the fluctuations of the output electric field in a quadrature defined by the phase angle θ . The corresponding fluctuation operator is written as:

$$\delta \hat{x}_\theta^{out}[\omega] = e^{-i\theta} \delta \hat{a}^{out}[\omega] + e^{i\theta} \delta \hat{a}^{out\dagger}[\omega]. \quad (22)$$

The noise spectrum for this quadrature is then given by

$$S_{\theta}^{out}[\omega] = S_{\hat{a}\hat{a}}^{out}[\omega] + S_{\hat{a}^\dagger\hat{a}^\dagger}^{out}[\omega] + e^{-2i\theta} S_{\hat{a}\hat{a}^\dagger}^{out}[\omega] + e^{2i\theta} S_{\hat{a}^\dagger\hat{a}}^{out}[\omega]. \quad (23)$$

The optimum squeezing spectrum for a given frequency is obtained for a quadrature θ_{opt} such that

$$e^{2i\theta_{opt}} = -\frac{S_{\hat{a}\hat{a}}^{out}[\omega]}{|S_{\hat{a}\hat{a}}^{out}[\omega]|} \quad (24)$$

which implies

$$S_{\theta_{opt}}^{out}[\omega] = 1 + 2[S_{\hat{a}\hat{a}^\dagger}^{out}[\omega] - |S_{\hat{a}\hat{a}}^{out}[\omega]|]. \quad (25)$$

We have calculated the curves giving the optimum noise spectrum, that is, for each given noise frequency, the noise of the quadrature that exhibits the best squeezing. Figure 3 shows the optimum noise at zero noise frequency as a function of the laser detuning $\delta = \delta_a = \delta_b$ from exciton and cavity resonance with the parameters of figure 1(a) ($g = 2 \times 10^{-2}$, $\gamma_a = \gamma_b = 0.25g$, $\alpha = 1.5 \times 10^{-9}$, $I_m = 1.6I_0$) and for mean numbers of phonons equal to 0 (zero temperature or no coupling with phonons), 0.5 and 1. It can be seen that a sizeable amount of squeezing is predicted in the absence of phonons when the exciting laser is resonant with the Rabi peaks. The squeezing vanishes quite rapidly as the dephasing effects, linked to the interaction with the phonons, come into play and excess noise appears at the same frequencies.

Figures 4 and 5 show the same noise spectra as in figure 3, in the very strong coupling regime, with the parameters of figures 2(a) and (b) respectively ($g = 2 \times 10^{-2}$, $\gamma_a = \gamma_b = 0.05g$, $\alpha = 1.5 \times 10^{-9}$, $I_m = 0.5I_0$ and $I_m = 2I_0$ respectively). The field in the cavity is of the same order of magnitude or smaller than in the previous figure. However, the reduced cavity width causes an enhancement of the nonlinear phase shift relative to the latter. The squeezing reaches 45% at zero temperature in the case of figure 5.

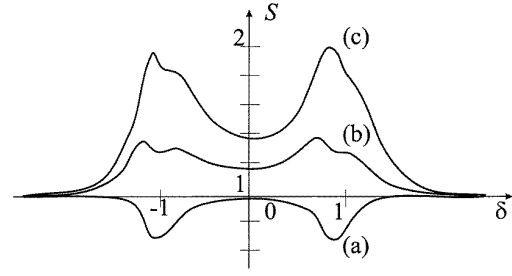


Figure 3. Optimum squeezing S (calculated at zero frequency) as a function of the laser detuning δ (normalized to g), for the parameters of figure 1(a). The standard quantum noise corresponds to $S = 1$, while perfect squeezing corresponds to $S = 0$. Curves (a), (b) and (c) correspond respectively to mean phonon numbers $\langle n \rangle = 0, 0.5$ and 1.

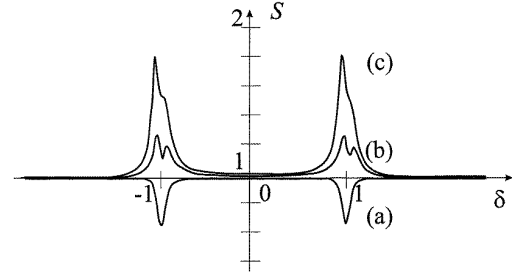


Figure 4. Optimum squeezing S (calculated at zero frequency) as a function of the laser detuning δ (normalized to g), for the parameters of figure 2(a). Curves (a), (b) and (c) correspond respectively to mean phonon numbers $\langle n \rangle = 0, 0.5$ and 1.

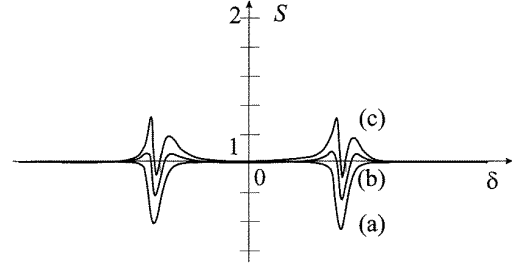


Figure 5. Optimum squeezing S (calculated at zero frequency) as a function of the laser detuning δ (normalized to g), for the parameters of figure 2(b). Curves (a), (b) and (c) correspond respectively to mean phonon numbers $\langle n \rangle = 0, 0.5$ and 1.

Another interesting feature is that the noise reduction is also less sensitive to the excess noise due to phonons and persists in the presence of small phonon numbers.

One can also plot the squeezing spectrum at a fixed laser frequency. The maximum values of squeezing are then obtained for noise frequencies close to those of the Rabi resonances. These values are similar to the ones shown in figures 3–5.

These curves have been calculated for excitonic densities of the order of 10^9 cm^{-2} , with one quantum well in the microcavity. With lower densities, the predicted squeezing scales down with the nonlinear phase shift. On the other hand, if one wants to increase the squeezing, it may be difficult to increase the excitonic density because density-dependent relaxation effects neglected here will come into play. By increasing the number of quantum wells in the cavity, it is,

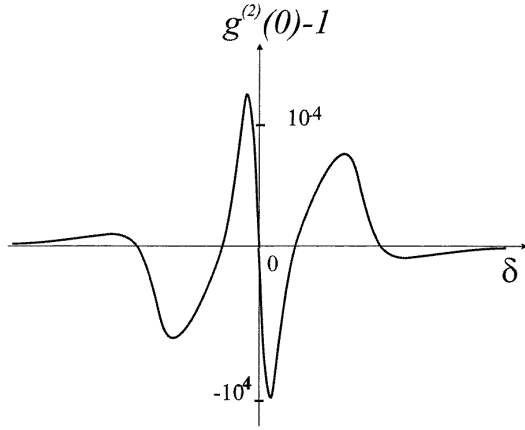


Figure 6. Intensity correlation function $g^{(2)}(0)$ for the output field as a function of the laser detuning δ (normalized to g). Parameters are the same as in figure 1(a).

however, possible to increase the nonlinear phase shift while keeping the excitonic density constant in each quantum well.

These results show that quantum effects should be observed in these semiconductor microcavities at low temperature or in a system decoupled from the phonons. The decoupling of the lower polariton branch from the phonons has been observed already in recent experiments [28, 36] and gives good prospect to such experiments.

5. Intensity correlation function

The intensity autocorrelation function is also a quantity of interest to explore QED effects in microcavities. Its expression is given by:

$$g^{(2)}(\tau) = \frac{\langle \hat{a}^\dagger(t) \hat{a}^\dagger(t+\tau) \hat{a}(t+\tau) \hat{a}(t) \rangle}{\langle \hat{a}^\dagger(t) \hat{a}(t) \rangle^2}. \quad (26)$$

It can be computed from the quantum fluctuations derived in the previous section by using the fact that the input noise is Gaussian. It is written as:

$$g^{(2)}(\tau) - 1 = \frac{2}{|a_0|^2} \left(C_{\hat{a}^\dagger \hat{a}}^{\text{out}}(\tau) + \text{Re} \left(\frac{a_0^2}{|a_0|^2} C_{\hat{a} \hat{a}}^{\text{out}}(\tau) \right) \right) \quad (27)$$

where the correlation functions as $C_{\hat{a} \hat{a}}^{\text{in}}(\tau)$ are the Fourier transform of the fluctuation spectra defined in equations (20):

$$C_{\hat{a} \hat{a}}^{\text{out}}(\tau) = \int \frac{d\omega}{2\pi} S_{\hat{a} \hat{a}}^{\text{out}}[\omega] e^{-i\omega\tau}. \quad (28)$$

Figure 6 shows the intensity correlation function at zero time delay in the absence of thermal noise. It can be seen that the system may exhibit bunching or antibunching depending on the pump laser detuning. The magnitude of the effect is small due to the fact that we assume a large number of photons in the field. It must be noted that, contrary to what is observed in single atomic systems (antibunching), we have a rich behaviour which is linked to the nature of the assumed nonlinearity.

We have also studied the time dependence of the intensity correlation function on τ . Figure 7 shows that $g^{(2)}(\tau)$ exhibits oscillations at the characteristic frequencies of the system.

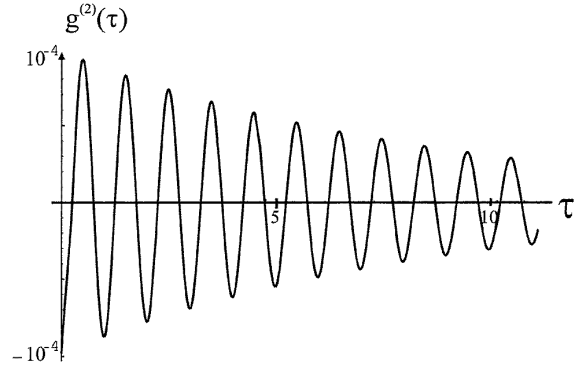


Figure 7. Intensity correlation function $g^{(2)}(\tau)$ for the output field as a function of τ (normalized to g^{-1}) for cavity–exciton resonance. Other parameters are the same as in figure 6.

In the case of very strong coupling, oscillations occur at the frequency g that is equal to the frequency of the oscillations in the transient luminescence of the system excited by a laser pulse in the same conditions. This behaviour is quite similar to the case of a microcavity containing atoms.

When thermal exciton noise is introduced, an interesting feature is obtained: the intensity correlation function is now always larger than one, that is in the classical domain. However, it retains the oscillation at the Rabi frequency. This oscillation, as well as the excess noise peaks described in the previous section, occurs at the resonance frequency of the system, as is well known in *noise spectroscopy* [37].

6. Conclusion

We have calculated the spectra of the quantum fluctuations of the field going out of a semiconductor microcavity and its intensity correlation function and shown that quantum effects such as squeezing and antibunching are predicted when thermal noise due to the coupling of excitons with phonons is very small. We have obtained significant squeezing even though the assumed nonlinearity is rather weak. When interaction with phonons is present, excess noise has been predicted to appear. The characteristics of this noise are linked to the dynamical behaviour of the excitons in the cavity.

The exploration of the noise should thus provide an interesting insight into the coherent and incoherent effects that are involved in the build-up and the destruction of quantum features. In order to predict the expected phenomena accurately, a more elaborate model for the relaxation of the polariton is needed to deal with the case in which the two polariton branches are coupled in a different way to the phonons. Nevertheless, we feel that the simple model presented here allows for identification of the main features of the quantum phenomena in semiconductor microcavities.

Acknowledgments

This work was supported by the EC ESPRIT LTR programme (contract no 20029 ACQUIRE) and the EC TMR programme (contract no ERBFMRXCT 96-00066).

References

- [1] Weisbuch C *et al* 1992 *Phys. Rev. Lett.* **69** 3314–17
- [2] Houdré R *et al* 1994 *Phys. Rev. Lett.* **73** 2043–6
- [3] Pau S, Björk G, Jacobson J, Cao H and Yamamoto Y 1995 *Phys. Rev. B* **51** 14 437
- [4] Chen Y, Treducci A and Bassani F 1995 *Phys. Rev. B* **52** 1800
- [5] Sermage B, Long S, Abram I, Marzin J Y, Bloch J, Planel R and Thierry-Mieg V 1996 *Phys. Rev. B* **53** 16 516
- [6] Haroche S 1992 *Cavity Quantum Electrodynamics Fundamental Systems in Quantum Optics* (Amsterdam: Elsevier) p 768
- [7] Meystre P *et al* 1990 *Opt. Commun.* **79** 300
- [8] Carmichael H J, Brecha R J and Rice P R 1991 *Opt. Commun.* **82** 73
- [9] Carmichael H J, Brecha R J, Raizen M G, Kimble H J and Rice P R 1989 *Phys. Rev. A* **40** 5516
- [10] Raizen M G, Thompson R J, Brecha R J, Kimble H J and Carmichael H J 1989 *Phys. Rev. Lett.* **63** 240
- [11] Rempe G, Thompson R J, Brecha R J, Lee W D and Kimble H J 1991 *Phys. Rev. Lett.* **67** 1727
- [12] Savona V, Hradil Z and Quattropani A 1994 *Phys. Rev. B* **49** 8774
- [13] Pau S, Björk G, Jacobson J, Cao H and Yamamoto Y 1995 *Phys. Rev. B* **51** 7090
- [14] Yamamoto Y, Matinaga F, Machida S, Karlsson A, Jacobson J, Björk G and Mukai T 1993 *J. Physique IV*, C5 **3** 39
- [15] Kaluzny Y, Goy P, Gross M, Raimond J M and Haroche S 1983 *Phys. Rev. Lett.* **51** 1175
- [16] Zhu Y, Gauthier D J, Morin S E, Wu Q, Carmichael H J and Mossberg T W 1994 *Phys. Rev. Lett.* **64** 2499
- [17] Orozco L A, Raizen M G, Xiao Min, Brecha R J and Kimble H J 1987 *J. Opt. Soc. Am. B* **4** 1490
- [18] Brune M, Schmidt-Kaler F, Maali A, Dreyer J, Hagley E, Raimond J M and Haroche S 1996 *Phys. Rev. Lett.* **76** 1800
- [19] Hanamura E 1974 *J. Phys. Soc. Japan* **37** 1545
- [20] Hanamura E 1974 *J. Phys. Soc. Japan* **37** 1553
- [21] Koch S W, Peygamberian N and Lindberg M 1988 *J. Phys. C: Solid State Phys.* **21** 5229
- [22] Mysyrowicz A, Hulin D, Antonetti A, Migus A, Masselink W T and Morkoc H 1986 *Phys. Rev. Lett.* **55**
- [23] Fox A M, Baumberg J J, Dabbicco M, Huttner B and Ryan J F 1995 *Phys. Rev. Lett.* **74** 1728
- [24] Eleuch H 1998 *Thèse Paris*
- [25] Agarwal G S and Puri 1989 *Phys. Rev. A* **39** 2969
- [26] Lugiato L and Strini G 1982 *Opt. Commun.* **41** 67
- [27] Castelli F, Lugiato L A and Vadicchino M 1988 *Nuovo Cimento B* **10** 183
- [28] Reid M D 1988 *Phys. Rev. A* **37** 4792
- [29] Hilico L, Fabre C, Reynaud S and Giacobino E 1992 *Phys. Rev. A* **46** 4397
- [30] Carmichael H J 1986 *Phys. Rev. A* **33** 3262
- [31] Steyn-Ross M L and Gardiner C W 1983 *Phys. Rev. A* **27** 310
- [32] Stanley R P, Pau S, Oesterle U, Houdré R and Ilegems M 1997 *Phys. Rev. B* **55** R4867
- [33] Tassone F, Piermarocchi C, Savona V, Quattropani A and Schwendimann P 1997 *Phys. Rev. B* **56** 7554
- [34] Janhke F *et al* 1996 *Phys. Rev. Lett.* **77** 5227
- [35] Piermarocchi C, Tassone F, Savona V, Quattropani A and Schwendimann P 1996 *Phys. Rev. B* **53** 15 584
- [36] Tassone F, Piermarocchi C, Savona V, Quattropani A and Schwendimann P 1996 *Phys. Rev. B* **53** R7642
- [37] Reynaud S and Heidmann A 1989 *Opt. Commun.* **71** 209
- [38] Collett M J and Gardiner C W 1984 *Phys. Rev. A* **30** 1386
- [39] Walls D F and Milburn G J 1994 *Quantum Optics* (Springer) pp 245–52
- [40] Houdré R, Gibernon J L, Pellandini P, Stanley R P, Oesterle U, Weisbuch C, O’Gorman J, Roycroft B and Ilegems M 1995 *Phys. Rev. B* **52** 7810
- [41] Nelson T R, Lindmark E K, Vick D V, Tai K, Khitrova G and Gibbs H M 1996 *Microcavities and Photonic Bandgaps* ed J Rarity and C Weisbuch (Dordrecht: Kluwer) p 43
- [42] Houdré R 1998 Private communication
- [43] Yabuzaki T, Mitsui T and Tanaka U 1991 *Phys. Rev. Lett.* **67** 2453